The $\Delta N \to NN$ transition in finite nuclei

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Abstract

We perform a direct finite nucleus calculation of the partial width of a bound Δ isobar decaying through the non-mesonic decay mode, $\Delta N \to NN$. This transition is modeled by the exchange of the long ranged π meson and the shorter ranged ρ meson. The contribution of this decay channel is found to be approximately 60 % of the decay width of the Δ particle in free space. Considering the additional pionic decay mode, we conclude that the total decay width of a bound Δ resonance in nuclei is of the order of 100 MeV and, consequently, no narrow Δ nuclear states exist, contrary to recent claims in the literature. Our results are in complete agreement with microscopic many-body calculations and phenomenological approaches performed in nuclear matter.

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I. INTRODUCTION

The $\Delta(1232)$ isobar is a well established nucleon resonance with spin-parity $J^\pi = \frac{3}{2}^+$ and isospin $I = \frac{3}{2}$. This resonance has a width of 120 MeV in free space from its strong decay to πN states. Pion- and photo- nuclear reactions at intermediate energies are dominated by the excitation of the Δ in the nucleus. Within the nuclear environment, the mesonic decay channel of a Δ gets reduced by the effect of Pauli blocking, since the outgoing nucleon finds difficulties in accessing nuclear unoccupied states. On the other hand, it is well known that a Δ in the nucleus can also decay through other mechanisms due precisely to the presence of the surrounding nucleons. Among those, the most quantitatively important channel being the one-nucleon induced process, $\Delta N \to NN$, which leaves nucleons with enough momentum and energy to overcome the Pauli blocking. This situation is analogue to what happens in hypernuclear decay, where a Λ particle bound in a nucleus of mass 5 and beyond decays predominantly through the weak $\Lambda N \to NN$ reaction. In the $\Delta N \to NN$ case, the reaction is strong, and thus, with a much larger signal than in the hypernuclear case.

In the late seventies and throughout the eighties, a lot of effort was dedicated to understand the properties of the Δ resonance in the nuclear medium in order to describe pion-nucleus scattering data [1, 2, 3, 4, 5, 6, 7, 8, 9]. Many analyses were made in terms of the Delta-hole model, which established an energy-dependent phenomenological Δ potential having an imaginary part of about -40 MeV at normal nuclear matter density [10, 11, 12, 13, 14]. Microscopic many-body calculations of the Δ width [15, 16, 17, 18, 19, 20, 21, 22], including the reduction of the pionic decay mode, $\Delta \to \pi N$, by the effect of Pauli blocking as well as its increase due to the new non-mesonic mode, $\Delta N \to NN$, and other pion absorption channels, obtain results that are in reasonable agreement with the phenomenological potential and, hence, describe satisfactory the pion-nucleus data. From all these studies it is safe to assert that the in-medium Δ width stays within the order of magnitude of the free width with a contribution from the $\Delta N \to NN$ channel of about 40–70 MeV.

Having apparently reached a satisfactory description of the Δ properties in a nuclear environment, the recent claims of the existence of narrow Δ states in nuclei come as a surprise [23]. The experiment was triggered by the apparent existence of narrow Σ states at CERN in spite of the strong $\Sigma N \to \Lambda N$ conversion mechanism [24, 25]. A recent experiment performed at Brookhaven with much better statistics did not observe narrow states for

targets of ^{6}Li and ^{9}Be in $(K^-, \pi^\pm)$ reactions, either for bound state or continuum regions [26], finalizing in this way many years of debate and speculation over possible mechanisms that could explain the existence of narrow Σ states in nuclei [27]. Nevertheless, some groups still claim the question to be unsettled due to the limited energy resolution of the recent experiments. The same groups advocate now the possibility of finding narrow Δ states in nuclei, even if the chances are *a priori* even worse than in the Σ case due to the existence of the strong pionic decay mode that is not completely blocked in a finite nucleus. These narrow Δ states in nuclei have also found some theoretical justification [28], the rationale being that most of the former theoretical works on the width of the Δ focused on a kinematical situation appropriate for pion-nucleus scattering at intermediate energies, where the Δ was created as a *quasifree* state. This favored the overlap of its wave-function with those of the emitted nucleons, hence producing large values for the decay width. However, the way the experiment of Ref. [23] was devised, measuring pions and protons in back-to-back coincidence, selects events in which the Δ is *bound* in a nucleus which, according to the theoretical model of Ref. [28], could not decay efficiently into two nucleons due to the little overlap between the initial and final wave-functions.

The purpose of the present work is to perform a direct finite nucleus calculation of the partial width of a *bound* Δ due to the non-mesonic mode $\Delta N \to NN$. Previous calculations of this decaying mode in a finite nucleus either concentrated on quasifree Δ states [16, 17, 21] or were actually induced from a finite-nucleus second order correction to nuclear matter amplitudes [22]. In order to account for all the physical ranges of the strong ΔN interaction, we use a one-pion plus one-rho exchange potential. While π-exchange is expected to describe reasonably well the long and intermediate ranges ($r_\pi \sim (m_\pi)^{-1} \sim (140)^{-1}$ MeV$^{-1} \sim 1.4$ fm), ρ-exchange ($m_\rho \sim 770$ MeV) is expected to cover shorter distances. The rest of the members of the pseudoscalar and vector octets (which would participate in a one-meson-exchange description of the process with masses up to ≈ 1 GeV) are not included here, since they are forbidden either by isospin conservation (η, ω), or by flavor conservation (K, K^*). For the description of the initial $^{12}_{\Delta}$C nucleus we use a shell model, where the Δ is assumed to weakly couple to a 11-particle core, from which we decouple the interacting nucleon, leaving a (properly antisymmetrized) spectator system of 10 nucleons. To account for the effects of the strong interaction in the initial two-body $(\Delta\, N)$ state, we multiply the corresponding (uncorrelated) two-body wave function, by an appropriate correlation function that takes

into account short-range repulsive effects phenomenologically. As for the final two-nucleon state, we solve a $T-$matrix scattering equation employing the Nijmegen nucleon-nucleon interaction.

The paper is organized as follows. In Sect. II we present the formalism which allows us to write the nuclear transition in terms of two-body matrix elements. In the same Section we explicitly write the most general form for these two-body amplitudes. In Sect. III we build up the regularized potential for the $\Delta N \to NN$ transition. In Sect. IV we present and discuss the results obtained, and in Sect. V we summarize our conclusions.

II. FORMALISM

A. Decay rate

In the center-of-mass (CM) frame, the decay rate of a nucleus due to the non-mesonic decay of a bound Δ resonance, $\Delta N \to NN$, is given by:

$$\Gamma_{\rm nm} = \int \frac{d^3P}{(2\pi)^3} \int \frac{d^3k}{(2\pi)^3} \overline{\sum} \, (2\pi) \, \delta(M_I - E_R - E_1 - E_2) \mid \mathcal{M}_{fi} \mid^2 \,, \tag{1}$$

where the initial bound system (${}^A_\Delta Z$ in what follows) has been considered to be at rest. The quantity $M_I = M(A-1,Z) + M_\Delta - B_\Delta$ is the mass of the initial nucleus with a bound Δ, with B_Δ being the Δ binding energy, while E_R, E_1 and E_2 are the energy of the residual $(A-2)-$particle system, and those of the two emitted nucleons, respectively. The integration variables $\vec{P}$ and $\vec{k}$ stand for the total and relative CM momenta of the two nucleons in the final state. The amplitude $\mathcal{M}_{fi} = \left\langle \Psi_R; \vec{P}\vec{k} \; s_1 \, s_2 \; t_1 \, t_2 \right| \hat{O}_{\Delta N \to NN} \left| {}^A_\Delta Z \right\rangle$ corresponds to the transition from an initial nuclear state containing a Δ particle, to a final state which is divided into two nucleons and a residual $(A-2)$-nucleon state, Ψ_R. The two-body operator responsible for this transition has been designated by $\hat{O}_{\Delta N \to NN}$. The $\overline{\sum}$ sum indicates an average over the projections (M_I) of the initial nucleus total spin (J_I) and a sum over all quantum numbers (J_R, M_R, T_R, T_{3_R}) of the residual $(A-2)-$particle system, as well as the spin (s_1, s_2) and isospin (t_1, t_2) projections of the emitted nucleons. We follow Ref. [29] and adopt a weak-coupling scheme where a Δ particle in an orbit $\alpha_\Delta = \{n_\Delta, l_\Delta, s_\Delta, j_\Delta, m_\Delta\}$ and charge t_{3_Δ} couples only to the ground-state wave function of

the nuclear $(A-1)$ core with quantum numbers J_C, M_C, T_C, T_{3_C}:

$$\mid \alpha_\Delta\rangle \otimes \mid A{-}1\rangle = \sum_{m_\Delta\, M_C} \langle j_\Delta m_\Delta\, J_C M_C \mid J_I M_I\rangle \mid (n_\Delta l_\Delta s_\Delta) j_\Delta m_\Delta\rangle \mid J_C M_C\rangle \mid t_\Delta t_{3_\Delta}\rangle \mid T_C T_{3_C}\rangle \,. \tag{2}$$

Employing the technique of the coefficients of fractional parentage, the core wave function is further decomposed into a set of states where the nucleon in an orbit $\alpha_N = \{n_N, l_N, s_N, j_N, m_N\}$ is coupled to a residual $(A-2)-$particle state:

$$\begin{aligned}
\mid J_C M_C\, T_C T_{3_C}\rangle &= \sum_{J_R T_R j_N} \langle J_C\, T_C \{\mid J_R\, T_R, j_N\, t_N\rangle [\mid J_R,\, T_R\rangle \times \mid (n_N l_N s_N) j_N,\, t_N\rangle]^{J_C M_C}_{T_C T_{3_C}} \\
&= \sum_{J_R T_R j_N} \langle J_C\, T_C \{\mid J_R\, T_R, j_N\, t_N\rangle \\
&\times \sum_{M_R\, m_N} \sum_{T_{3_R}\, t_{3_i}} \langle J_R M_R\, j_N m_N \mid J_C M_C\rangle \langle T_R T_{3_R}\, t_N t_{3_i} \mid T_C T_{3_C}\rangle \\
&\times \mid J_R M_R\rangle \mid T_R T_{3_R}\rangle \mid (n_N l_N s_N) j_N m_N\rangle \mid t_N t_{3_i}\rangle \,,
\end{aligned} \tag{3}$$

where $t_N = \frac{1}{2}$. The appropriate spectroscopic factors $S = (A-1)\, \langle J_C T_C \{\mid J_R T_R, j_N t_N\rangle^2$ in the case of ${}^{12}_{\Delta}\mathrm{C}$ are taken from Ref. [30].

Assuming the Δ to decay from a $l_\Delta = 0$ orbit, focusing only on the processes induced by the neutral Δ^0 ($t_\Delta = \frac{3}{2}$, $t_{3_\Delta} = -\frac{1}{2}$), and working in a coupled two-body spin and isospin basis, the nonmesonic decay rate in Eq. (1) can be written as:

$$\Gamma_{\rm nm} = \Gamma_{\rm n} + \Gamma_{\rm p} \,, \tag{4}$$

with $\Gamma_{\rm n}$ and $\Gamma_{\rm p}$ the neutron- (Δn $\to$ nn) and proton-induced (Δp $\to$ np) decay rates, respectively, given by:

$$\begin{aligned}
\Gamma_i \; &= \int \frac{d^3 P}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3}\, (2\pi)\, \delta(M_I - E_R - E_1 - E_2) \sum_{S M_S} \sum_{J_R M_R} \sum_{T_R T_{3_R}} \frac{1}{2J_I + 1} \\
&\times \sum_{M_I} \mid \langle T_R T_{3_R} \frac{1}{2} t_{3_i} \mid T_C T_{3_C}\rangle \mid^2 \\
&\times \Bigg| \sum_{T T_3} \langle T T_3 \mid \frac{1}{2} t_1\, \frac{1}{2} t_2\rangle \sum_{m_\Delta M_C} \langle j_\Delta m_\Delta\, J_C M_C \mid J_I M_I\rangle \sum_{j_N} \sqrt{S\,(J_C\, T_C\,; J_R\, T_R\,, j_N\, t_{3i})} \\
&\times \sum_{M_R m_N} \langle J_R M_R\, j_N m_N \mid J_C M_C\rangle \sum_{m_{l_N} m_{s_N}} \langle j_N m_N \mid l_N m_{l_N} \frac{1}{2} m_{s_N}\rangle \\
&\times \sum_{S_0 M_{S_0}} \langle S_0 M_{S_0} \mid \frac{3}{2} m_\Delta\, \frac{1}{2} m_{s_N}\rangle \sum_{T_0 T_{3_0}} \langle T_0 T_{3_0} \mid \frac{3}{2}\; -\frac{1}{2}\;\; \frac{1}{2} t_{3_i}\rangle \\
&\times\; t_{\Delta N \to NN}(S, M_S, T, T_3, S_0, M_{S_0}, T_0, T_{3_0}, l_\Delta, l_N, \vec{P}, \vec{k}) \Bigg|^2 \,, \qquad i = {\rm n}, {\rm p}
\end{aligned} \tag{5}$$

where $t_{\Delta N \to NN}$ is the elementary $\Delta N \to NN$ transition amplitude in the nucleus.

B. Two-Body Amplitudes

In this section we derive the elementary two-body transition amplitude, $t_{\Delta N \to NN}$, which describes the one-nucleon induced decay of the Δ-particle in nuclei.

First, we need to write the product of two single-particle wave functions, $\langle \vec{r}_1 \mid \alpha_\Delta \rangle$ and $\langle \vec{r}_2 \mid \alpha_N \rangle$, in terms of relative and center-of-mass coordinates, $\vec{r}$ and $\vec{R}$. Using the Moshinsky brackets [31] one may connect the wave-functions for two particles in a common harmonic oscillator (H.O.) potential with the wave-function given in terms of the relative and center-of-mass coordinates of the two particles. In the present work, the single-particle Δ and N orbits are taken to be solutions of harmonic oscillator mean field potentials with parameters b_Δ and b_N respectively. Their values are found by using the single-particle energies of the Δ particle given in Ref. [23] on the one hand, and by fitting the charge form factor of ^{12}C on the other hand. The values thus obtained are $b_\Delta = 1.59$ fm and $b_N = 1.64$ fm respectively. We assume the Δ binding energy to be given approximately by the s-shell energy of the mean-field model used in Ref. [23], namely $B_\Delta = -\varepsilon_{s_{\frac{3}{2}}} \sim 25$ MeV.

Assuming an average size parameter $b = (b_\Delta + b_N)/2$ and working in the LS representation, the product of the two harmonic oscillator single-particle states, $\Phi^{\Delta}_{nlm}(\vec{r}_1\,/b)$ and $\Phi^{N}_{n'l'm'}(\vec{r}_2\,/b)$, can be transformed to a linear combination of products of relative and center-of-mass wave functions, $\Phi^{\rm rel}_{N_r L_r M_{L_r}}(\vec{r}\,/\sqrt{2}b)$ and $\Phi^{\rm CM}_{N_R L_R M_{L_R}}(\vec{R}\,/(b/\sqrt{2}))$, respectively. Since the Δ is in a $l_\Delta = 0$ shell, one obtains:

$$\Phi^{\Delta}_{100}\left(\frac{\vec{r}_1}{b}\right) \Phi^{N}_{100}\left(\frac{\vec{r}_2}{b}\right) = \Phi^{\rm rel}_{100}\left(\frac{\vec{r}}{\sqrt{2}b}\right) \Phi^{\rm CM}_{100}\left(\frac{\vec{R}}{b/\sqrt{2}}\right) , \tag{6}$$

when the nucleon is in the s-shell and

$$\begin{aligned} &\Phi^{\Delta}_{100}\left(\frac{\vec{r}_1}{b}\right) \Phi^{N}_{11m}\left(\frac{\vec{r}_2}{b}\right) = \\ &\frac{1}{\sqrt{2}} \left\{ \Phi^{\rm rel}_{100}\left(\frac{\vec{r}}{\sqrt{2}b}\right) \Phi^{\rm CM}_{11m}\left(\frac{\vec{R}}{b/\sqrt{2}}\right) - \Phi^{\rm rel}_{11m}\left(\frac{\vec{r}}{\sqrt{2}b}\right) \Phi^{\rm CM}_{100}\left(\frac{\vec{R}}{b/\sqrt{2}}\right) \right\} \end{aligned} \tag{7}$$

when the nucleon is in the p-shell. With this decomposition, the amplitude $t_{\Delta N \to NN}$ of Eq. (5) can be written in terms of amplitudes which depend on C.M. and relative orbital angular momentum quantum numbers

$$t_{\Delta N \to NN} \;=\; \sum_{N_r L_r N_R L_R} X(N_r L_r N_R L_R, l_\Delta l_N)\; t^{N_r L_r\, N_R L_R}_{\Delta N \to NN} \;, \tag{8}$$

where $X(N_r L_r N_R L_R, l_\Delta l_N)$ are the Moshinsky brackets, which for $l_\Delta{=}l_N{=}0$ are just $X(1\,0\,1\,0,\ 0\,0) = 1$, and for $l_N = 1$ are $X(1\,0\,1\,1,\ 0\,1) = 1/\sqrt{2}$ and $X(1\,1\,1\,0,\ 0\,1) = -1/\sqrt{2}$.

As for the final NN state, the antisymmetric state of two independently moving nucleons with total momentum $\vec{P}$ and relative momentum $\vec{k}$ reads:

$$\langle \vec{R}\,\vec{r} \mid \vec{P}\,\vec{k}\ S\,M_S\ T\,M_T\rangle = \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\vec{P}\,\vec{R}}\left(\mathrm{e}^{\mathrm{i}\vec{k}\,\vec{r}} - (-1)^{S+T}\mathrm{e}^{-\mathrm{i}\vec{k}\,\vec{r}}\right)\chi^S_{M_S}\chi^T_{M_T}\ . \tag{9}$$

In order to incorporate the effects of the NN interaction, the plane wave describing the relative NN motion needs to be substituted by a distorted wave, $\mathrm{e}^{\mathrm{i}\vec{k}\,\vec{r}} \rightarrow \Psi_{\vec{k}}(\vec{r})$, solution of a $T-$matrix scattering equation, with the input of appropriate and realistic baryon-baryon potentials. The formalism to derive these distorted wave functions is described in great detail in Ref. [32]. In the present work we strictly follow such formalism while using the Nijmegen Soft-Core NSC97f strong potential model [33] in the calculation. The limited knowledge of the $\Delta N \rightarrow \Delta N$ interaction, essentially due to the unknown $\Delta\Delta$-meson vertices, has led us to treat the short-range ΔN effects phenomenologically via the correlation function

$$f(r) = \left(1 - \mathrm{e}^{-(r^2/a^2)}\right)^2 + br^2\mathrm{e}^{-(r^2/c^2)}\ , \tag{10}$$

where $a = 0.5$ fm, $b = 0.25$ fm^{-2} and $c = 1.28$ fm. We have checked that our results are rather insensitive to the particular shape and strength of this correlation function, once realistic NN wave-functions are incorporated.

The matrix elements $t^{N_r L_r\, N_R L_R}_{\Delta N \rightarrow NN}$ in Eq. (8) are then given by:

$$\begin{aligned} t^{N_r L_r\, N_R L_R}_{\Delta N \rightarrow NN} &= \frac{1}{\sqrt{2}}\int d^3R\ \Phi^{\mathrm{CM}}_{N_R L_R}\left(\frac{\vec{R}}{b/\sqrt{2}}\right)\mathrm{e}^{-\mathrm{i}\vec{P}\,\vec{R}} \\ &\times \int d^3r\,\chi^{\dagger\,S}_{M_S}\chi^{\dagger\,T}_{T_3}\,\Psi^*_{\vec{k}}(\vec{r})\,V_{\sigma\tau}(\vec{r})\,f(r)\Phi^{\mathrm{rel}}_{N_r L_r}\left(\frac{\vec{r}}{\sqrt{2}b}\right)\chi^{S_0}_{M_{S_0}}\chi^{T_0}_{T_{3_0}} \\ &= (2\pi)^{3/2}\,\Phi^{CM}_{N_R L_R}\left(\vec{P}\frac{b}{\sqrt{2}}\right)\ t_{\mathrm{rel}}\ , \end{aligned} \tag{11}$$

with

$$t_{\mathrm{rel}} = \frac{1}{\sqrt{2}}\int d^3r\,\chi^{\dagger\,S}_{M_S}\chi^{\dagger\,T}_{T_3}\,\Psi^*_{\vec{k}}(\vec{r})\,V_{\sigma\tau}(\vec{r})\,f(r)\Phi^{\mathrm{rel}}_{N_r L_r}\left(\frac{\vec{r}}{\sqrt{2}b}\right)\chi^{S_0}_{M_{S_0}}\chi^{T_0}_{T_{3_0}}\ , \tag{12}$$

where, for simplicity, only the direct amplitude –first term of Eq. (9)– is shown.

In the next section, we show how the potential $V_{\sigma\tau}(\vec{r})$ can be decomposed as:

$$V_{\sigma\tau}(\vec{r})\ =\ \sum_i\sum_\alpha V^{(i)}_\alpha(r)\hat{O}_\alpha\,\hat{I} = \sum_i\{V^{(i)}_{SS}(r)\vec{S}_1\vec{\sigma}_2 + V^{(i)}_T(r)S_{12}(\hat{r})\}\,\hat{I}, \tag{13}$$

where the index i runs over the different mesons exchanged (π and ρ), and α over the different spin operators, $\hat{O}_\alpha \in (\ \vec{S}_1\, \vec{\sigma}_2, \ S_{12}(\hat{r}) \equiv 3\, \vec{S}_1\, \hat{r}\, \vec{\sigma}_2\, \hat{r} - \vec{S}_1\, \vec{\sigma}_2)$, written in terms of the spin $\frac{3}{2} \to \frac{1}{2}$ transition operator $\vec{S}$ and the spin Pauli matrices $\vec{\sigma}$. Since both mesons have isospin 1, the isospin operator, $\hat{I}$, factorizes out of the sum in $V_{\sigma\tau}(\vec{r})$. This operator takes the form $\vec{T}_1\vec{\tau}_2$, with $\vec{T}$ and $\vec{\tau}$ having the same structure as $\vec{S}$ and $\vec{\sigma}$, respectively. It can be shown that this operator only connects isotriplet ΔN and NN states.

By performing a partial-wave expansion of the final two-nucleon wave-function and working in the $(LS)J$-coupling scheme, the relative $\Delta N \to NN$ amplitude, $t_{\rm rel}$, can be further decomposed:

$$
\begin{aligned}
t_{\rm rel} \ &= \ \frac{1}{\sqrt{2}} \sum_{i\,\alpha} \sum_{LL'J} 4\pi {\rm i}^{-L'} \ \langle L M_L S M_S | J M_J \rangle \ Y_{LM_L}(\hat{k}_r) \\
& \times \ \langle L_r M_{L_r} S_0 M_{S_0} | J M_J \rangle \ \langle (L'S) J M_J \mid \hat{O}_\alpha \mid (L_r S_0) J M_J \rangle \\
& \times \ \langle T T_3 \mid \hat{I} \mid T_0 T_{3_0} \rangle \int r^2 dr\, \Psi^{*\,J}_{LL'}(k_r, r)\, V^{(i)}_\alpha(r)\, f(r)\, \Phi^{\rm rel}_{N_r L_r}(\frac{r}{\sqrt{2}b}) \ . \qquad (14)
\end{aligned}
$$

The explicit expressions for the expectation values of the spin dependent operator, $\langle (L'S) J M_J \mid \hat{O}_\alpha \mid (L_r S_0) J M_J \rangle$, can be found in the Appendix.

Taking into account all the possible initial states, the required antisymmetry of the NN wave function and the couplings induced by the NN strong interactions, the allowed $\Delta N \to NN$ transitions are:

$$
\begin{aligned}
&{}^3P_0 \ \to \ {}^3P_0 \\
&{}^3P_1 \ \to \ {}^3P_1 \\
&{}^3P_2 \ \to \ {}^3P_2, \ {}^3F_2 \\
&{}^5S_2 \ \to \ {}^1D_2
\end{aligned}
$$

III. THE MESON-EXCHANGE POTENTIAL

As we already mentioned, we assume that the $\Delta N \to NN$ transition, depicted in Fig. 1, proceeds via the exchange of the virtual π and ρ mesons. The Lagrangians entering each vertex are [34]:

$$\mathcal{L}_{NN\pi} = \frac{f_{NN\pi}}{m_\pi} \bar{\Psi} \gamma^5 \gamma^\mu \vec{\tau} \Psi \partial_\mu \vec{\Phi}_\pi \ , \qquad (15)$$

$$\mathcal{L}_{N\Delta\pi} = \frac{f_{N\Delta\pi}}{m_\pi} \bar{\Psi} \vec{T} \Psi_\mu \partial^\mu \vec{\Phi}_\pi \ , \qquad (16)$$

$$\mathcal{L}_{NN\rho} = g_\rho \bar{\Psi} \gamma_\nu \vec{\tau} \Psi \vec{\Phi}^\nu_\rho + \frac{f_\rho}{2M} \bar{\Psi} \sigma_{\mu\nu} \vec{\tau} \Psi \partial^\mu \vec{\Phi}^\nu_\rho \ , \qquad (17)$$

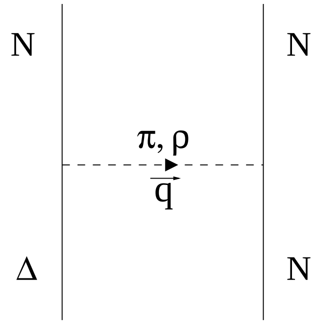

FIG. 1: Feynman diagram for the exchange of π and ρ mesons in the $\Delta N \rightarrow NN$ transition.

and

$$\mathcal{L}_{N\Delta\rho} = \mathrm{i}\frac{f_{N\Delta\rho}}{m_\rho}\bar{\Psi}\gamma^5\gamma_\nu\vec{T}\Psi_\mu(\partial^\nu\vec{\Phi}^\mu_\rho - \partial^\mu\vec{\Phi}^\nu_\rho)\,. \tag{18}$$

where Ψ_μ is the Rarita-Schwinger field operator describing the Δ-isobar and M is the nucleon mass.

The nonrelativistic reduction of the free space Feynman amplitude is associated with the transition potential. In momentum space the one pion exchange potential takes the form [see Ref. [15]]:

$$V^\pi(\vec{q}\,) = \frac{g_{NN\pi}}{2M}\frac{g_{N\Delta\pi}}{2\overline{M}_{\Delta N}}\frac{(\vec{S}_1\,\vec{q}\,)\,(\vec{\sigma}_2\,\vec{q}\,)}{\vec{q}^{\,2}+m_\pi^2-q_0^2}\;\vec{T}_1\cdot\vec{\tau}_2\;, \tag{19}$$

where $\vec{q}$ is the momentum carried by the pion directed towards the $NN\pi$ vertex, q_0 its energy (which cannot be neglected due to the $m_\Delta - m_N$ difference) and $\overline{M}_{\Delta N}$ the average between the nucleon and Δ masses. We have introduced the coupling constants $g_{NN\pi}$ and $g_{N\Delta\pi}$ that relate to those in the Lagrangians through $f_{NN\pi}/m_\pi = g_{NN\pi}/(2M)$ and $f_{N\Delta\pi}/m_\pi = g_{N\Delta\pi}/(2\overline{M}_{\Delta N})$, respectively.

For the ρ meson the potential takes the form:

$$V^\rho(\vec{q}\,) = \frac{(g_\rho+f_\rho)}{2M}\frac{g_{N\Delta\rho}}{2\overline{M}_{\Delta N}}\frac{(\vec{S}_1\times\vec{q}\,)\,(\vec{\sigma}_2\times\vec{q}\,)}{\vec{q}^{\,2}+m_\rho^2-q_0^2}\;\vec{T}_1\cdot\vec{\tau}_2\;, \tag{20}$$

where $g_{N\Delta\rho}$ is defined through the relation $f_{N\Delta\rho}/m_\rho = g_{N\Delta\rho}/(2\overline{M}_{\Delta N})$. Performing a Fourier transform of the general expression given in Eq. (19) and Eq. (20), using the relation $(\vec{S}_1\times\vec{q}\,)(\vec{\sigma}_2\times\vec{q}\,) = (\vec{S}_1\vec{\sigma}_2)\vec{q}^{\,2} - (\vec{S}_1\vec{q}\,)(\vec{\sigma}_2\vec{q}\,) = \frac{2}{3}(\vec{S}_1\vec{\sigma}_2)\vec{q}^{\,2} - \frac{1}{3}S_{12}(\hat{q})\vec{q}^{\,2}$, it is easy to obtain the corresponding transition potential in coordinate space, which can be divided into central (SS) and tensor (T) pieces that take the following form:

$$V^{(i)}_{SS}(r) \;=\; K^{(i)}_{SS}\frac{1}{3}\left[(m_i^2-q_0^2)\frac{e^{\mathrm{i}\sqrt{q_0^2-m_i^2}\,r}}{4\pi r} - \delta(r)\right] \equiv K^{(i)}_{SS}V_{SS}(r,m_i)\;, \tag{21}$$

TABLE I: Expressions for the $K_\alpha^{(i)}$ constants entering the π and ρ potentials. The values of the strong coupling constants and cutoffs are taken from Ref. [33].

Meson	$K_{SS}^{(i)}$	$K_T^{(i)}$	Strong CC	Λ_i (GeV)
π	$\frac{g_{N\Delta\pi}}{2\overline{M}_{\Delta N}}\frac{g_{NN\pi}}{2M}$	$\frac{g_{N\Delta\pi}}{2\overline{M}_{\Delta N}}\frac{g_{NN\pi}}{2M}$	$g_{NN\pi}=13.16$	1.3
			$g_{N\Delta\pi}=32.4$	
ρ	$2\frac{g_{N\Delta\rho}}{2\overline{M}_{\Delta N}}\frac{f_\rho+g_\rho}{2M}$	$-\frac{g_{N\Delta\rho}}{2\overline{M}_{\Delta N}}\frac{f_\rho+g_\rho}{2M}$	$g_{N\Delta\rho}=38.1$	1.4
			$f_\rho=12.52$	
			$g_\rho=2.97$	

$$V_T^{(i)}(r) \;=\; K_T^{(i)}\frac{1}{3}(m_i^2-q_0^2)\frac{e^{\mathrm{i}\sqrt{q_0^2-m_i^2}\,r}}{4\pi r}\left(1+\frac{3}{\mathrm{i}\sqrt{q_0^2-m_i^2}r}-\frac{3}{(q_0^2-m_i^2)r^2}\right)\equiv K_T^{(i)}V_T(r,m_i)\;,$$

with $i=\pi$ or ρ. In order to account for the finite size of the particles, we use a monopole form factor $F_i(\vec{q}^{\,2})=(\Lambda_i^2-m_i^2)/(\Lambda_i^2+\vec{q}^{\,2}-q_0^2)$ at each vertex, where the value of the cut-off, Λ_i, depends on the meson. The use of form factors leads to the following regularized potential for each meson:

$$\begin{aligned}V_{\mathrm{SS}}(r;m_i) \;&\to\; V_{\mathrm{SS}}(r;m_i)-V_{\mathrm{SS}}(r;\Lambda_i)-\frac{1}{2}(\Lambda_i^2-m_i^2)\sqrt{\Lambda_i^2-q_0^2}\,\mathrm{e}^{-\sqrt{\Lambda_i^2-q_0^2}\,r}\left(1-\frac{2}{\sqrt{\Lambda_i^2-q_0^2}r}\right)\;,\\ V_{\mathrm{T}}(r;m_i) \;&\to\; V_{\mathrm{T}}(r;m_i)-V_{\mathrm{T}}(r;\Lambda_i)-\frac{1}{2}(\Lambda_i^2-m_i^2)\sqrt{\Lambda_i^2-q_0^2}\,\mathrm{e}^{-\sqrt{\Lambda_i^2-q_0^2}\,r}\left(1+\frac{1}{\sqrt{\Lambda_i^2-q_0^2}r}\right)\;.\end{aligned} \tag{22}$$

In Table I we show the explicit expressions for the $K_\alpha^{(i)}$ coefficients, as well as the values of the strong coupling constants and cutoffs used in this work. Note that we use the phenomenological value $\frac{f_{N\Delta\pi}}{f_{NN\pi}}=2.13$ which reproduces the free Δ width. This same ratio is applied to obtain the coupling strength of the Δ to the ρ meson from that of the nucleon through the relation:

$$\frac{f_{N\Delta\rho}}{m_\rho}=\left(\frac{f_{N\Delta\pi}}{f_{NN\pi}}\right)\frac{g_\rho+f_\rho}{2M}. \tag{23}$$

IV. RESULTS

Our results for the total decay rates are presented in Table II in units of the Δ decay width in free space, $\Gamma_\Delta = 120$ MeV, for the exchange of a π meson, a ρ meson and the combination of both. The different columns show results without strong short range correlations (label *free*), with only initial ΔN interactions (label *ISI*), and including, in addition, the final NN interactions (label *ISI+FSI*) through the corresponding strongly correlated wave functions for the initial $\Delta\, N$ and final NN states. We also show separately the contribution coming from the decay induced by a nucleon in the $l_N = 0$ shell (label s), by a nucleon in the $l_N = 1$ shell (label p) and the sum of both contributions (label s+p).

TABLE II: Decay width of the Δ isobar when the non-mesonic decay is modeled by the exchange of a π meson, a ρ meson and both mesons, $\pi + \rho$. The results in the different columns correspond to not considering strong short range correlations (*free*), including only the initial ΔN correlations (*ISI*) or the combined effect of initial and final state interactions (*ISI + FSI*). Values are given in units of the free Δ decay width, $\Gamma_\Delta = 120$ MeV.

shell	meson	*free*	*ISI*	*ISI+FSI*
s	π	0.29	0.29	0.28
	ρ	0.22×10^{-2}	0.64×10^{-4}	0.19×10^{-3}
	$\pi + \rho$	0.35	0.29	0.27
p	π	0.34	0.34	0.33
	ρ	0.48×10^{-2}	0.39×10^{-3}	0.63×10^{-3}
	$\pi + \rho$	0.40	0.33	0.31
$s + p$	π	0.63	0.63	0.61
	ρ	0.69×10^{-2}	0.45×10^{-3}	0.83×10^{-3}
	$\pi + \rho$	0.74	0.61	0.58

The results of Table II show that the decay width is dominated by the $\pi-$exchange mechanism. The $\rho-$exchange contribution is very small although it interferes in a non-negligible way with $\pi-$exchange. We also notice that short-range correlations modify very moderately the $\pi-$exchange decay width, representing a 3% effect when both initial and final correlations are included. Correlations affect in a much more relevant way the $\rho-$exchange process,

reducing the corresponding rate by roughly a factor of 10. In general, when introduced independently, initial or final correlations reduce the total $\pi + \rho$ decay rate by about 15%, while their combined effect lowers the rate by 20%.

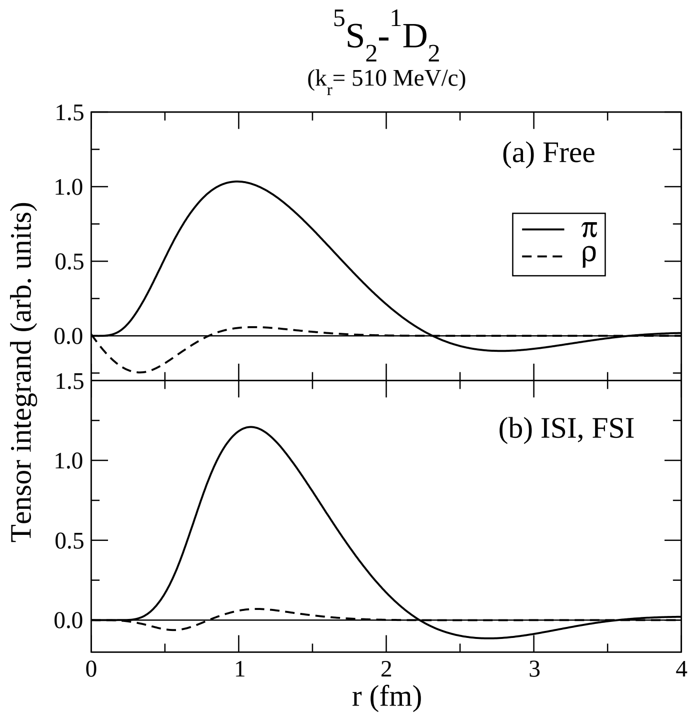

FIG. 2: π (solid line) and ρ (dashed line) contributions to the integrand of the dominant $^5S_2 \rightarrow ^1 D_2$ transition amplitude as a function of the relative distance r. The upper panel displays the free amplitudes, while the initial ΔN (*ISI*) and final NN (*FSI*) short-range effects are included in the amplitudes of the lower panel.

The systematics observed in the results presented in Table II can be better understood by examining Fig. 2. There, the integrand of the tensor transition amplitude in Eq. (14), which gives the most important contribution to the Δ width as we will see, is shown as a function of the relative distance r, for a representative relative NN momentum value of $|\vec{k}_r| = 510$ MeV/c and in the case of the $\pi-$exchange (solid lines) and the ρ-exchange (dashed lines) mechanisms. All the results include form factors at the vertices, but the amplitudes in the upper panel are obtained using uncorrelated ΔN and NN wave functions, while both initial and final correlations are included in the results of the lower panel. One clearly sees that, being of long range nature, the pion-exchange amplitude is little affected by the inclusion of short-range correlations. In contrast, the $\rho-$exchange potential, being more short ranged

due to its larger meson mass, peaks at much shorter distances, hence the implementation of strongly correlated wave functions in the lower panel reduces the integrand substantially. In addition, the $\rho-$exchange potential shows a node in the space region of relevance, which induces an additional cancellation of the $\rho-$contribution once *ISI+FSI* effects are included.

The contribution of the tensor transition potential to the decay width dominates by two orders of magnitude that of the central term, as we can see in Table III. This is essentially due to the fermionic character of the final two-nucleon state which requires the NN wave function to be antisymmetric, *i.e.*, the relative NN quantum numbers have to verify the $L+S+T=$odd relation, where L, S and T stand for angular momentum, spin, and isospin respectively. Since T is necessarily 1 and the central spin operator connects only states with $S_0 = S = 1$, the orbital angular momentum L of the final NN must be odd. This can only be achieved from an initial ΔN pair having $L_r = 1$, which means that the central transition amplitude induced by s-shell nucleons is exactly zero since we consider the Δ to be in the lowest energy state, $1s_{\frac{1}{2}}$. For p-shell nucleons the central transition is not forbidden, since 50% of the wave function has a relative angular momentum $L_r = 1$, but even in this case the tensor transitions dominate due to the large momentum transferred in the reaction ($|\vec{q}| \sim 500$ MeV/c), which favors also a large amount of angular momentum transfer.

TABLE III: Decay width of the Δ isobar when the non-mesonic decay is modeled by the exchange of a $\pi+\rho$ mesons considering initial and final state interactions (*ISI* + *FSI*). The different contributions of the central and tensor channels are presented and their contributions to the diferents shells. Values are given in units of the free Δ decay width, $\Gamma_\Delta = 120$ MeV.

shell	Central	Tensor	TOTAL
s	0	0.27	0.27
p	$0.16\ \times 10^{-2}$	0.31	0.31
$s+p$	$0.16\ \times 10^{-2}$	0.58	0.58

The calculated in-medium width of the Δ due to the non-mesonic $\Delta N \rightarrow NN$ mechanism represents a 58% fraction of the free width, i.e. it amounts to $\Gamma_\Delta = 70$ MeV. In the optical potential language this would correspond to an imaginary part of about $\mathrm{Im}\, U_\Delta = \Gamma_\Delta/2 =$ 35 MeV, in perfect agreement with extrapolations at zero momentum of the absorptive

optical potential, calculated for quasifree Δ's in Ref. [20], and with the phenomenological analysis in various nuclei [10, 11, 12, 13, 14].

Our result is about 50% larger than the finite nucleus calculation of Ref. [22], where the effect of NN correlations is accounted for via a nuclear-matter G-matrix while direct ΔN short range correlations are ignored. The origin of the discrepancy comes essentially from the use of a phenomenological $N\Delta\pi$ coupling constant here, with a value $f_{\pi N\Delta}/f_{\pi NN} = 2.13$ adjusted to reproduce the free Δ width, which is 25% larger than the the quark model value $f_{\pi N\Delta}/f_{\pi NN} = 6\sqrt{2}/5 = 1.70$ employed in Ref. [22].

Finally, we also note that the contribution to the Δ width from the $\Delta N \to NN$ mechanism explored here is in excellent agreement with the analogous nucleon pole contribution calculated by Oset and Salcedo [19] in nuclear matter for a pion kinetic energy of $T_\pi \sim 100$ MeV, which, in their notation, would correspond to a Δ binding energy of 25 MeV, as assumed in the present work, although with a finite momentum of 150–200 MeV/c.

V. CONCLUSIONS

Motivated by recent speculations on the possible existence of narrow Δ-nuclear states, we have performed the first direct finite nucleus calculation of the partial width of a *bound* Δ resonance via the decay mechanism $\Delta N \to NN$, including ΔN correlations and realistic NN interactions. We find that, in ${}^{12}_{\Delta}\mathrm{C}$, this partial width represents a 58% fraction of the free width, i.e. it amounts to $\Gamma_\Delta = 70$ MeV.

Our result, evaluated explicitly for a Δ nuclear bound state, is in quantitative agreement with extrapolations to low momentum of the partial widths obtained for Δ quasifree states in nuclear matter.

Considering also the partial width of the mesonic mode $\Delta \to N\pi$, which can be quenched by about 50% in nuclei due to Pauli blocking, we therefore conclude that the total decay width of a *bound* Δ resonance in nuclei is of the order of 100 MeV and, consequently, narrow Δ states cannot be formed in finite nuclei.

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APPENDIX

In this Appendix, the explicit expressions for the $\langle (L'S)JM_J|\hat{O}_\alpha|(L_rS_0)JM_J\rangle$ coefficients appearing in the evaluation of the relative $\Delta N \to NN$ amplitude will be given. The quantum numbers L_r, S_0, J and M_J stand for the initial relative orbital angular momentum, the initial coupled intrinsic spin and the total spin and spin projection of the ΔN state, while the numbers L', S, J and M_J are the pertinent quantities for the final NN system.

A. Spin-Spin Transition

$$\begin{aligned} &\hat{O}_\alpha = \vec{S}_1\vec{\sigma}_2 \\ &\langle (L'S)JM_J|\hat{O}_\alpha|(L_rS_0)JM_J\rangle = -\frac{4}{\sqrt{6}}\ \delta_{L_rL'}\delta_{S_0S}\delta_{S1} \end{aligned} \tag{A.2}$$

B. Tensor transition

$$\hat{O}_\alpha = S_{12}(\hat{r}) = 3\vec{S}_1\hat{r}\vec{\sigma}_2\hat{r} - \vec{S}_1\vec{\sigma}_2 \tag{A.3}$$

The tensor operator only allows for $S_0 = 2 \to S = 0$ and $S_0 = 1 \to S = 1$ transitions, with matrix elements:

$S_0 = 2 \to S = 0$	$L_r = J+2$	$L_r = J+1$	$L_r = J$	$L_r = J-1$	$L_r = J-2$
$L' = J$	$-3\sqrt{\frac{(J+1)(J+2)}{(2J+1)(2J+3)}}$	0	$\sqrt{\frac{6J(J+1)}{(2J-1)(2J+3)}}$	0	$-3\sqrt{\frac{J(J-1)}{(2J+1)(2J-1)}}$

$S_0 = 1 \to S = 1$	$L_r = J+1$	$L_r = J$	$L_r = J-1$
$L' = J+1$	$-\frac{J+2}{\sqrt{6}(2J+1)}$	0	$\frac{3}{2J+1}\sqrt{\frac{J(J+1)}{6}}$
$L' = J$	0	$\frac{1}{\sqrt{6}}$	0
$L' = J-1$	$\frac{1}{2J+1}\sqrt{\frac{3}{2}J(J+1)}$	0	$-\frac{1}{\sqrt{6}}\left(\frac{J-1}{2J+1}\right)$

C. Isospin matrix elements

$$
\begin{aligned}
&\hat{I} = \vec{T}_1\vec{\tau}_2 \\
&\langle TT_3|\hat{I}|T_0T_{30}\rangle = -\frac{4}{\sqrt{6}}\ \delta_{T_3T_{30}}\delta_{T_0T}\delta_{T1}
\end{aligned}
\tag{A.4}
$$

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